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Welded Continuous Frames and Their Components

Column Research Council Project D.2.D

COLUMNS IN CONTINUOUS FRAMES

By

Lynn S. Beedle

These are the notes written following
presentation of a description of the
Lehigh program at the Technical Session
of the Column Research Council

May 11, 1951

Fritz Engineering Laboratory
Department of Civil Engineering and Mech.

Lehigh University

FL Report No. 205A.5

SLIDE LIST - 205A.5

(For Report No 205A.5)

LINE NO.	FINAL NO.	OLD PHOTO NO. FILE NO.	OLD SLIDE NO.	SUBJECT	STATUS		
					TO DO	DONE	DONE
1	14	19		<u>COLUMNS</u> - SEE LINE 35	X	X	X
2	1	Tab. I	205A.2-3	TEST PROGRAM AND LOAD CONDITIONS (234.6A)	X	X	X
3	3	4	205A.2-4	Method of applying Loads	X	X	X
4	4	17	205A.2-15	Int. curve, "b" 8WF31 - Test Results (2)	X	X	X
5	2	2	205A.2-2	Previous Methods of test (Disc.)	X	X	X
6	A	16	205A.2-B	RESTRAINT - Box culvert	X	X	X
7	13	P1205	-	8WF31 - 16' - "C" - lateral buckling	X	X	X
8	7	4:22		Influence of Loading condition	X	X	X
9	6	4:21		Influence of Slenderness Ratio	X	X	X
10	8	4:19		3-dimensional Int. curve	X	X	X
11	12	2:18		Carry-over factor 4WF13	X	X	X
12	5	4:7+8		Moment due to bending + due to axial load	X	X	X
13	10	1:7		Load condition C 8WF31	X	X	X
14	9	1:5		" " b[a,d] 8WF31	X	X	X
15	11	1:6		" " b 4WF13	X	X	X
16				Moment-angle change relations			
17							
18							
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28							
29	20	P128		<u>LOCAL BUCKLING</u> T1, T2: buckling at knee	X	X	X
30	19	P125		(Frames) T1, T2: uniform moment	X	X	X
31	18	P219		Frame T2 - General view	X	X	X
32		P711	205B.1-G	<u>LOCAL BUCKLING</u> 14WF30 Support prior to collapse	X	X	X
33		P703	205B.1-16	" " buckled	X	X	X
34			205.2-5	8WF40 Support	X	X	X
35	14		205.2-8	8WF31 Column --- buckled	X	X	X
36	13	51	205C.1-12	M- ϕ_{AVE} for all 14 connections	X	X	X
37		79	205C.1-17	Buckling of conn. L (8B) - 8B13 shape	X	X	X
38							
39							
40	-	16	205A.1-	<u>STRESS-STRAIN</u> T.M. - Stress curve	X	X	X
41	D		205.3-14	8WF40 - Strain-hard. (comp.)	X	X	X
42	C		205.3-1	"7 FUNCTIONS" - Initial σ - ϵ DIAG.	X	X	X
43	-		205.3-3	SIMPLIFIED σ - ϵ	X	X	X
44	E		205.3-18	8WF40 - T&C - 10 curves	X	X	X
45	-		205.3-7	Initial σ - ϵ - Various struc. Stls	X	X	X

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INTRODUCTION

In 1946 the AISC commenced sponsorship at Lehigh University under the direction of Dr. Bruce G. Johnston the current investigation of columns loaded with combined axial force and end moment.

The objectives at that time were:

- (a) to determine the plastic behavior of steel columns of the type used in building frames with continuous connections, and
- (b) it was also sought at that time to determine carry-over and stiffness factors.

The program is now part of a larger investigation sponsored by WRC, and ONR with funds furnished by the AISC, AISI, BUSHIPS, BUY&D, ONR.

I represent a considerable number of people who have worked on this project, commenced by Dr. Johnston. Joseph Ready, Mr. C.H. Chen, Mr. Jan Ruzek and Robert Ketter have in that sequence been Research Assistants on the project. Mr. Ketter is now actively carrying out the investigation.

Both analytical and experimental phases of the investigation are proceeding ~~in parallel~~ simultaneously. In this brief presentation, the experimental attack will be described, followed by a discussion of the theory. The comparison between theory and the experiments carried out up to the present time is given and certain appropriate conclusions are drawn.

EXPERIMENTAL EFFORT

The load-carrying capacity of steel columns is of course a function of many variables. SLIDE 1 shows a program of tests and indicates the variables being examined in this program:

- (a) Loading condition (5 are being studied)
- (b) Size effect (4WF13 and 8WF31)
- (c) Flexure Axis
- (d) Slenderness Ratio (27 to 120)
- (e) Magnitude of Axial Load (ratio of P to M)
- (f) End Condition
- (g) Shape of cross-section.

This latter variable is not evident in the slide, since the two sections have almost identical shapes. At some future time some experiments should be carried out with this variable in mind.

Some of the methods that have been used by previous investigators to attack some of the above problems are shown in SLIDE NO. 2

In each of these it is either difficult or impossible to independently vary axial load and end moment....mentioned above as one of the unique features of the investigation.

A second feature is, of course, the size of the member, since commercially-available as-delivered columns are being tested in this program.

SLIDE 3 shows the test apparatus being used in the experimental program at Lehigh University. The test column is welded to a base plate at each end, these plates in turn being bolted to end fixtures with moment arms.

Axial load is applied with a screw-type machine of 800,000# capacity. The end bending moments are applied with the help of a frame shown in the slide as a "test frame". Forces applied hydraulically by pumps and jacks deliver the ~~same~~ moment to column ends. By virtue of the adjustable cross-beams the test frame carries the moment.

Horizontal ties carry the shear force. By reversing either the direction of the moment arm or the position of the jack and dynamometer assembly the directions of end moments may be reversed. In those tests where a "fixed-end" condition is desired, this is accomplished with the aid of level bars.

Among the test methods possible, two have been used. In the one case, the full magnitude of axial load has been applied, and subsequently bending moments have been applied to collapse. In a few cases the reverse procedure has been used.

ANALYSIS AND THEORY

Considering sketch (a) on the blackboard, when no end bending moment is applied the member is a column whose ultimate strength is predicted by the tangent modulus theory with satisfactory engineering accuracy. This critical load is P_E for an elastic column or P_y for an inelastic column if the idealized stress-strain diagram of sketch (b) is assumed resulting in the column curve of sketch (d).

On the other hand, as is evident in sketch (d) for zero axial load the member is a beam with the predicted M_y and M_p being shown.

If now these two curves are put together then for certain combinations of axial load and bending moment there will be a locus of points of initial yield and a locus of points for collapse.

SLIDE 4 shows such an "interaction curve". Two curves are obtained (as already mentioned). For this particular case the initial yield condition is given by

$$P/A + Mc/I = f_y$$

The method of test is also indicated. The collapse curve is also shown and the curve is derived on the basis of the theory outlined by the second set of sketches on the blackboard.

(discussed formation of elastic limit and collapse curves)

SLIDE 5 shows the influence of axial load. When L/r is zero (or approaches zero) the influence of axial load is only to decrease in linear proportion the stress at which the yield point is reached in bending.

For L/r not zero under combined bending and axial load the moment at a section will be made up of two parts: that due to moment and that due to axial load.

In the first combined diagram the critical section for yielding is at the end, the load P being less than P_L to be defined. As the axial load is increased there will be reached a load at which the moment diagram indicates zero slope at the end. This axial load is called P_L , the limit axial load for this condition. As the load is increased further the point of maximum moment (and thus maximum stress) ~~xxx~~ moves away from the end and occurs at some interior point.

The phenomenon just described is a function of L/r , since, for a more slender column it is logical that this ~~xxxxxxxxxxxx~~ P_L value be lowered. It is also a function of load condition, for "c" there would never be a moment maximum at the ends.

Interaction curves are drawn throughout this investigation in terms of M_o , the moment at the end of the column.

These curves show the influence of slenderness ratio (SLIDE 6). Notice for L/r of 112 the successive decrease in capacity (initial yield) as the load condition is changed from a-b-d-c.

SLIDE 7 shows ~~xxx~~ somewhat more clearly the influence of condition of loading on the initial yield strength. For one particular member the allowable from the AISC formula is also shown in this slide. For the different load conditions the P_L value is reached as follows:

- a: P_E
- b: $.55P_E$
- c: 0
- d: $.27P_E$

A three-dimensional interaction curve (developed in this program at Lehigh) is shown in ~~Fig~~ SLIDE 8. By adding a z-coordinate with units of in/in a more complete picture of theoretical column behavior is obtained. In the load-L/r coordinate system we see the familiar column curve (plotted of course to the opposite hand ~~from~~ to that which we are accustomed). In the horizontal plane is the familiar lateral buckling curve for members loaded in bending only. The interaction curves are evident in the third coordinate system.

The surface in space traced out by the initial yield and collapse conditions may be grasped.

The AISC formula is again shown as a basis for comparison.

One point of interest is the plane surface generated by the initial yield condition. If confirmed experimentally this might eliminate the possibility of separate consideration of L/r.....for a particular range.

COMPARISON OF THEORY AND EXPERIMENT

Formulas have been developed for the various load conditions to predict the initial yield strength. Other theories are partially developed for the collapse condition.

SLIDE 9 compares experiment with theory for load condition b, 8WF31, $L/r = 56$, $KL/r = 0.7 \times 56$. Observation of the first yield line is shown by the short line. The circles indicate "yield strength" according to the BJ criterion and the maximum strength is indicated by the squares.

Tests 3 and 4 develop the strength. T5 failed by lateral collapse and the evidence indicates that this is due to residual stress. A residual stress level of about 12 ksi is evident from the curves.

Additional tests at low P/P_{cr} were conducted (13 and 14) for load conditions d and a respectively. No difficulty was experienced in reaching the predicted values. In the case of 14, two tests were run, the column being tested again under higher axial load after completing the lower point.

Strain-hardening probably accounts for a considerable portion of the increase above the predicted collapse value.

Tc

SLIDE 10 shows the other extreme in loading condition.... the same L/r , an 8" column, but loaded in single curvature.

The column does not develop even its elastic limit strength, let alone the predicted collapse value. Again, the formation of ~~initial yield line~~ first yield line, the development of Yield Strength, and the collapse points are indicated.

T12 shows a reduction of 25% alone due to load condition. (Compare with T4 on Slide 9).

The Jezek solution is plotted. There is a range, of course, where the result is impossible, indicating collapse below the predicted yield point. The same solution has been attempted on 4WF13 members but the results are impossible and indicate that real study needs to be given the factor which depends upon cross-sectional form in the Jezek procedure.

This slide also shows the other method of test used.

SLIDE 11 is for load condition b, the four tests indicating the influence of slenderness ratio. For the increased slenderness ratio of 112 (double that of slide 9) the column has further difficulty in developing its yield strength.

None of these columns tested developed the predicted collapse strength according to the plastic theory due to collapse by combined bending and twist.

For these proportions, restraint, and loading, the theoretical maximum load ~~for the pure end~~ with zero end moment is 104 kips.

T0 and T7 gave identical results, T0 having been initially deformed plastically under axial load alone. T7 was a "pure" test. The influence of the prior plastic deformation was negligible.

SLIDE 12 plots a stress function ~~for~~ against carry-over factor. The theoretical elastic value for this particular axial load is given by ~~the~~ the dotted line. The observed points are shown by the circles. The dot-dash line gives the ratio of end angle changes for load condition d.... valid only to the elastic limit.

Stiffness and carry-over factors are being determined in all the tests. While of value to confirm our theories in the elastic range, the value of the plastic results has not yet been established.

Earlier was mentioned the mode of failure of some of the columns tested. Some of these will now be pictured.

SLIDE 13 shows T12 which failed by combined bending and twist. This is typical of all condition "c" tests and for those columns which do not develop their predicted ~~yield strength~~ collapse strength. L/P for this test is 56. The moment is not a maximum at the end. VI

SLIDE 14 shows the type of failure that occurs when the columns do successfully carry the predicted strength. In this case the strength of the column is finally limited by local buckling of flange elements which occurs well after the column has reached the yield condition. VI

SLIDES 15, 16, and 17 show local buckling of beams and of a connections

The frame of SLIDE 18 was tested under 3/8-point loading. The condition of the central span is shown in SLIDE 19 and of the connections in SLIDE 20. The other members in each case are 8WF40, the frame previously shown having been 8B13 shape. The influence of local buckling in this case is quite severe.

SUMMARY AND DISCUSSION

1. In single curvature the column does not develop its yield strength unless axial load is relatively low. VI
2. Residual stress accounts for some of the reduction in load capacity
3. There appears to be a residual stress level of about 12 ksi in the column material tested (8").
4. Whenever the moment is not a maximum at the end then the column is in a more serious condition.
5. For other load conditions than single curvature the column either will or will not develop the predicted collapse strength depending on L/P and P .
6. So long as axial load is such that the maximum moment is at the end, then the column will at least develop its yield strength.

ADDITIONAL NOTES NOT INCLUDED IN TALK

The purpose of this study is to determine the plastic behavior of columns. Columns in buildings are occasionally restrained columns and the program treated thus far in this talk has not dealt with this subject. However, the work is important since it provides an indication of the ability of columns to carry moments assumed. I

For example the tests show that, if these were members in a truss, bent in the same direction, as that used in these tests, the influence of secondary bending stresses may be quite pronounced. VI

Since these columns will not even carry condition "c" loading to the predicted yield point, it is extremely doubtful whether or not the strengths predicted by the Winter-Bijlaard theory may be realized.

ACKNOWLEDGEMENTS AND REFS

The assistance of various individuals indicated earlier in the text.

This material has been taken in part from:

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S-3
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o T-20
T-11

→ R-26
→ R-27
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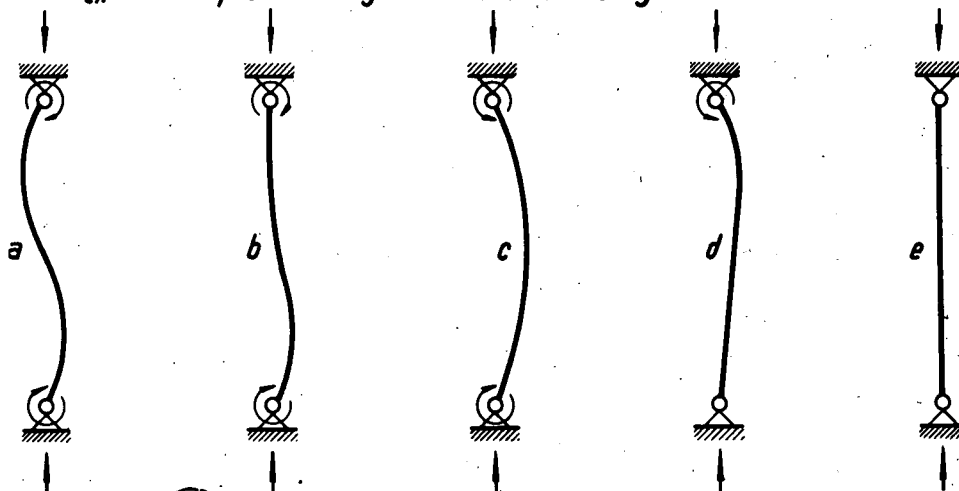
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PROPOSED COLUMN TESTS

Tests	Size of member	Axis	l	r	l/r	Test conditions
Pilot	4 WF13	X-X	16	1.72	111.5	e, d, b
5 Various P/P_{cr} ①	4 WF13	X-X	8 12 16	1.72	56.0 83.8 115.5	a, b, c a, b, c, splice a, b, c, d
"	4 WF13	Y-Y	8	0.99	97.1	a, b, c
"	8 WF31	X-X	8 12 16	3.47	27.6 41.6 52.0	a, b, c a, b, c, splice a, b, c, d
"	8 WF31	Y-Y	8	2.11	47	a, b, c
"	4 WF13 8 WF31	45°	8 8	-	-	b, splice

① P_{cr} = load predicted by Tan. modulus theory.



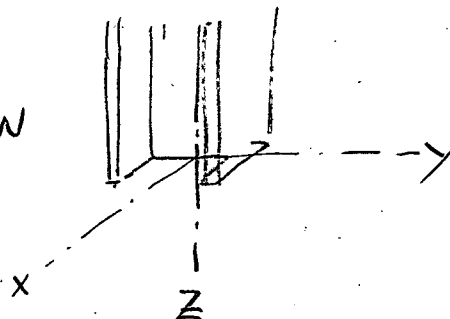
①

CONDITION OF TEST

TABLE I

⑥

END CONDITION



TESTS OF COLUMNS UNDER COMBINED THRUST AND MOMENT *

by LYNN S. BEEDLE, JOSEPH A. READY, and BRUCE G. JOHNSTON
Fritz Engineering Laboratory, Department of Civil Engineering
and Mechanics, Lehigh University, Bethlehem, Pa.

ABSTRACT

Under sponsorship of the American Institute of Steel Construction, test equipment has been developed in which various desired combinations of axial load and end moments may be applied to metal columns. Lengths of 8, 12, and 16 feet may be accommodated in the apparatus, which is designed to test steel columns up to an 8WF40 rolled section size.

Axial load is applied by a universal testing machine of 800,000 pound capacity. The end moments are applied separately through lever arms with tension-compression hydraulic jacks mounted in series with dynamometers which measure the thrust.

The behavior of test columns under load is determined by the use of four measuring techniques: (1) taut-wire and mirror deflection gages, (2) level bars, (3) SR-4 strain gages, and (4) whitewash. In addition, the moment-producing thrust is measured by aluminum tube dynamometers.

The test program is currently being sponsored by the Welding Research Council. Some of the test results obtained are presented to demonstrate the effectiveness of the apparatus. The objectives of the program are described, the principal one being to determine the ultimate strength of steel columns under varying combinations of end moments and direct load.

INTRODUCTION

For a number of years the American Institute of Steel Construction has sponsored research on the general column problem at the Fritz Engineering Laboratory of the Lehigh University Civil Engineering Department. A study of the local buckling of the flanges of WF columns was completed in 1942.⁽¹⁾ This was followed by an investigation of the behavior of eccentrically-loaded columns.⁽²⁾ The previous work contains rather complete references to other theoretical and experimental studies of the column problem.

In 1946, the AISC commenced sponsorship of

the current investigation of columns loaded with combined axial force and various end moments, simulating the loads acting on a column in a rigid frame. Two principal objectives are:

(1) to determine the ultimate strength of columns under combined axial load and end moment, and

(2) to determine the moment distribution carry-over and stiffness factors of steel columns in both the elastic and plastic range. A variation in the magnitude of compressive load will influence these factors.

Some similar tests have been conducted on small specimens. So far as is known to the authors, none approaching column sizes used in structures have been made in which a constant axial load could be maintained while the applied moment was varied, or in which the applied moment could be maintained constant during change in axial load. These, then, are the essential features of this investigation:

(a) the use of rolled structural steel members, and

(b) the ability to independently vary axial load and end moment.

The program is now coordinated with a five-year investigation of the ultimate strength of welded continuous frames sponsored by the Welding Research Council with financial support from the American Institute of Steel Construction, The American Iron and Steel Institute, the U. S. Navy Bureau of Yards and Docks, and the U. S. Navy Bureau of Ships through a contract with the Office of Naval Research. The Column Research Council also supports the work in an advisory capacity.

In Fig. 1 are shown the forces applied by the test apparatus used in the current investigation. It will be seen that it is possible to apply axial load to the test specimen independently of end moments. The concentric force, P , is applied through knife edges with an 800,000 lb. Riehle testing machine. Thrusts, F , applied hydraulically to lever arms, deliver the moment. Lateral support, H , is provided. Previous investigators have used some of the testing arrangements shown in Fig. 2, since their study was

*Progress Report No. 2 on the "Ultimate Strength of Welded Continuous Frames and Their Components"

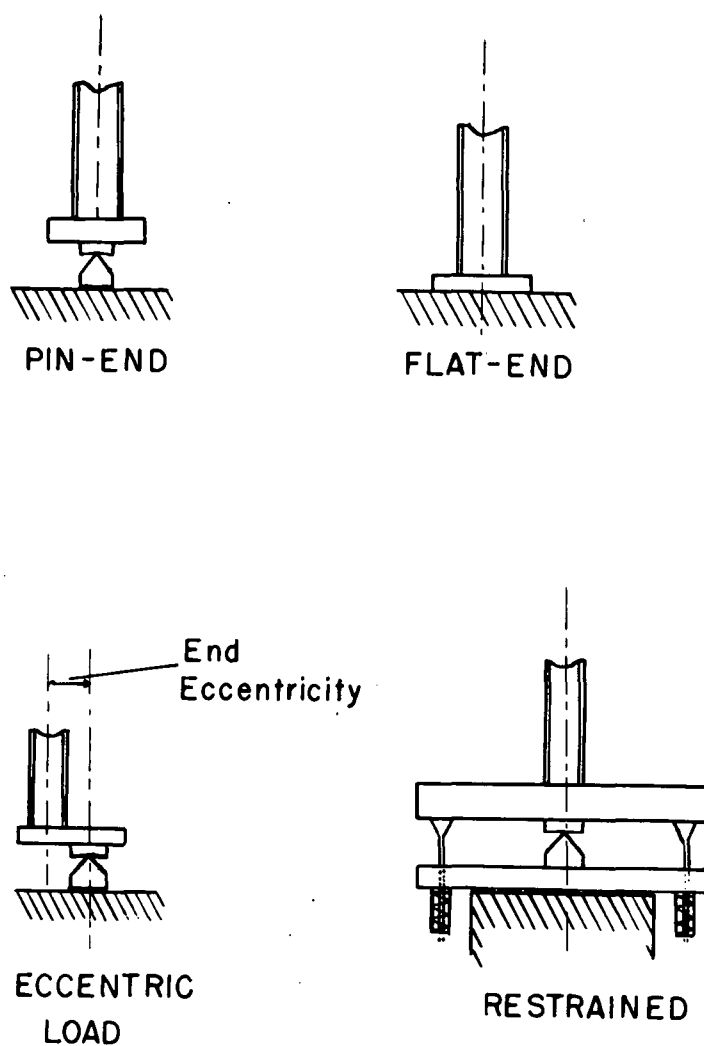


FIG. 2 - COLUMN TEST METHODS
USED BY PREVIOUS INVESTIGATORS

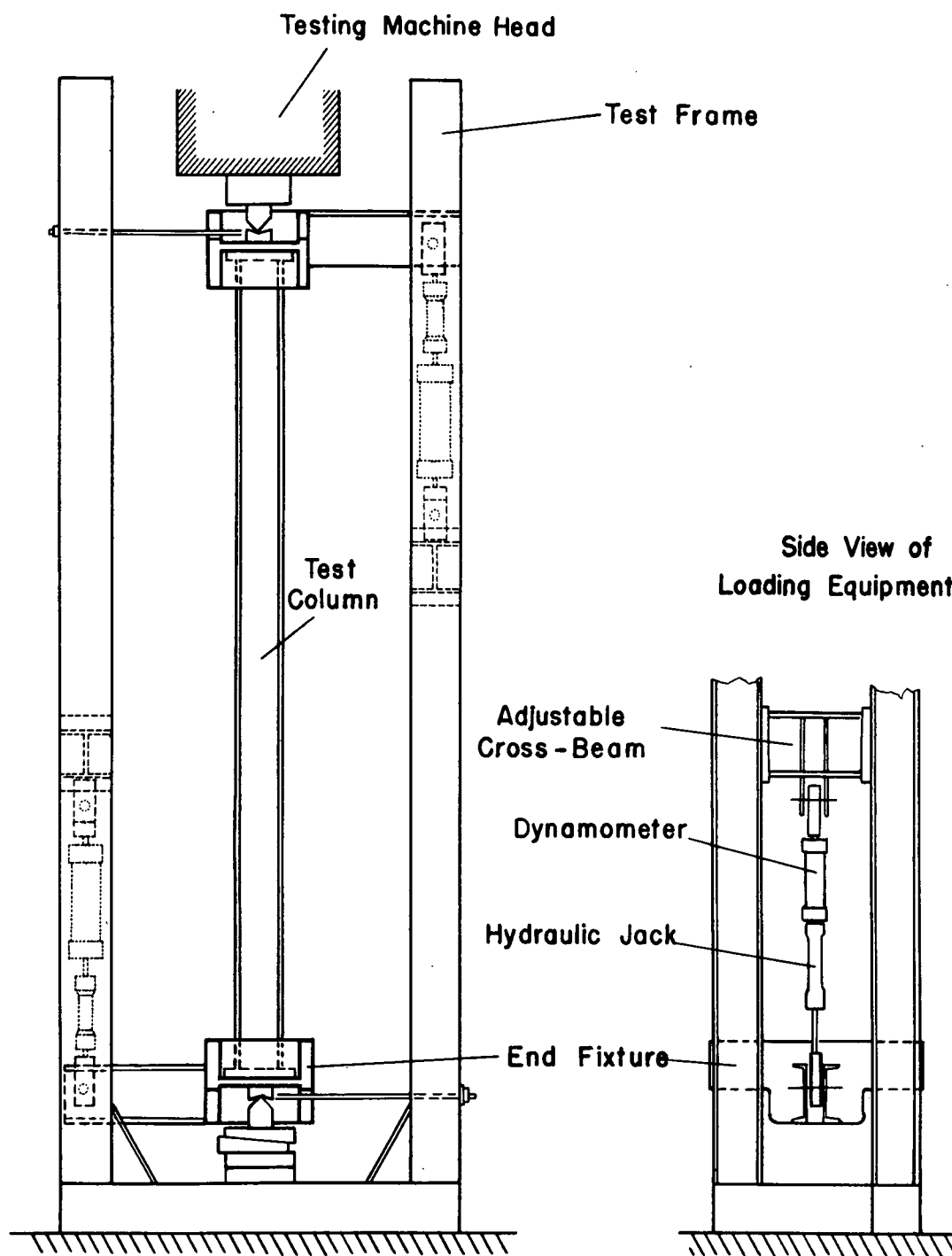


FIG. 4 - COLUMN TEST APPARATUS

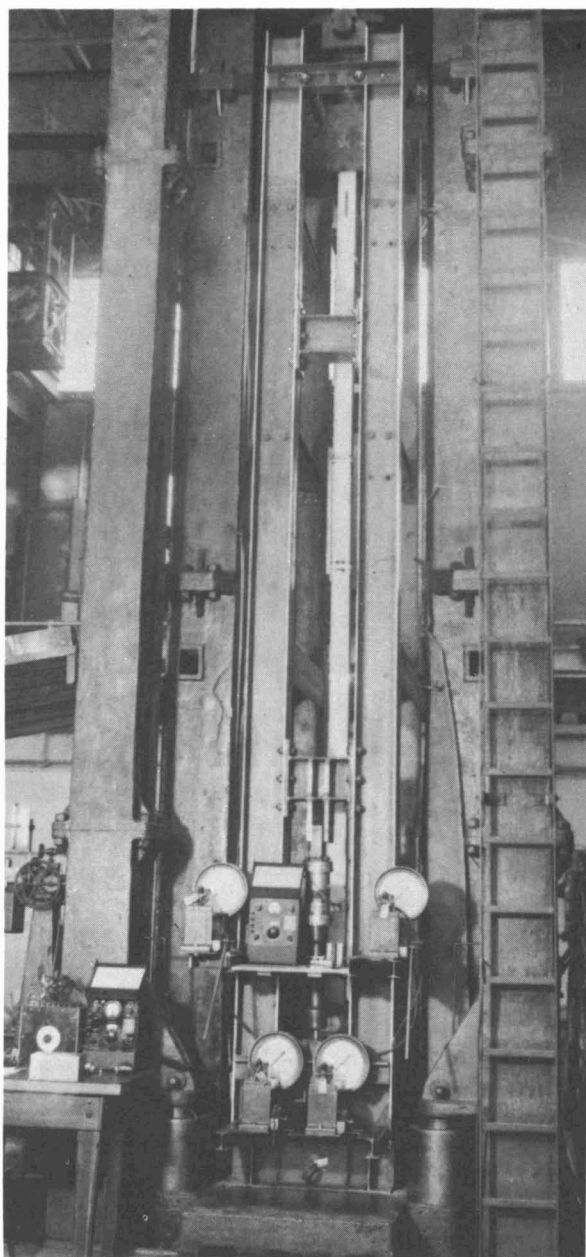


Fig. 5. Frame and column under load in testing machine

column are separately centered between the vertical screws of the testing machine.

The knife edge seats, being rigidly attached to the end fixtures, act to automatically position the knife edge blocks which are supported with wedge blocks and cylindrical bearings, enabling them to shift to the proper position. The moment arm is centered in the test frame

by adjustment of lateral tie rods.

Thus with the equipment described, the column is placed squarely in line with the testing machine axis, and the moment arm is positioned in the intended plane of bending, normal to the axis of knife edges.

Prior to an actual test, the alignment is checked using SR-4 gages and level bars at the ends. The SR-4 gages indicate the eccentricity at the ends (Fig. 7 is a typical arrangement), whereas the measured rotations tend to average all the eccentricities and constitute a more general indication of alignment. Thus far in the investigation, centering under load, (the process by which the column is shifted with respect to the machine screws or supports) has been used to a limited extent.* Extreme accuracy of alignment of the vertical axis is not necessary since a moment of considerable magnitude is applied at one or both ends in all tests.

Hydraulic Pumps, Jacks, and Dynamometers

As previously mentioned, the application of end moment is accomplished with tension-compression jacks in series with load measuring dynamometers. Two pumps are connected with high pressure tubing to each jack, one for use in tension, the other for use in compression.** The pumps, high pressure tubing and jacks were procured commercially***, the dynamometers having been designed and constructed at the Fritz Engineering Laboratory making use of SR-4 gages under license from Baldwin-Southwark Corporation. Fig. 6 shows the system in operation in the pilot test.

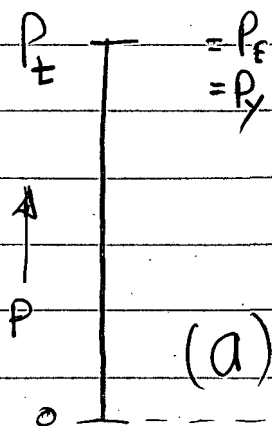
The dynamometers shown in the photographs consist of aluminum tubes with heavy ends threaded to receive the jack and pin connectors. Four SR-4 strain gages, type AD-1, were installed on each unit.

The arrangement of strain gages on the dynamometers is one in which both the active, "A", and compensating, "D", gages are mounted on the aluminum tubing, the "D" gages being

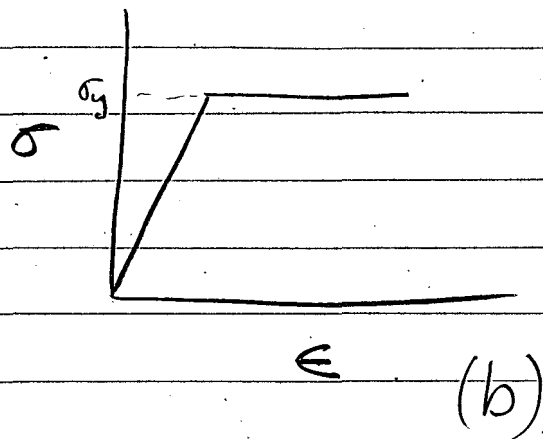
* The use of this process by others has been described by Osgood.⁽⁴⁾

** In order to use the system at high compressive loads it is necessary to provide lateral support to the jacks. Thus far it has been possible to run tests using tension alone.

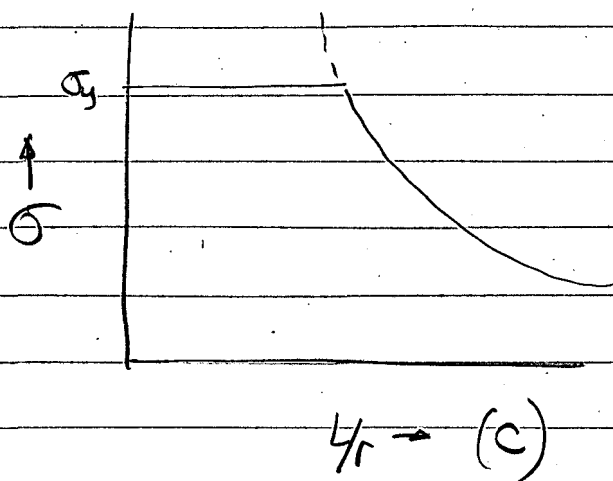
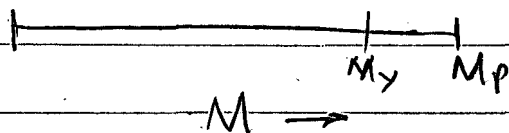
***Messinger Bearing Co. Philadelphia, Pa.



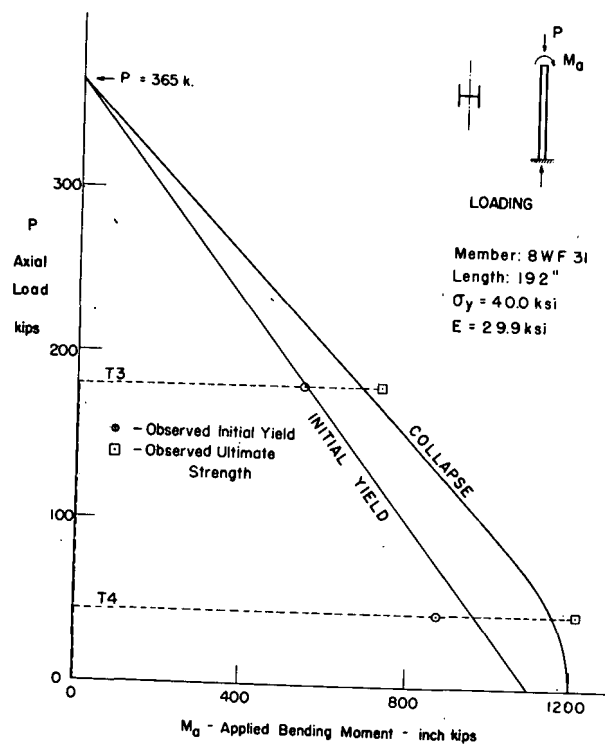
(a)

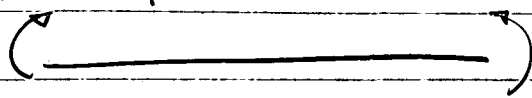


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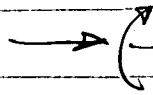
 $L_r \rightarrow$ (c)

(d)

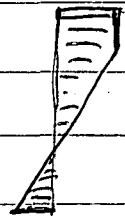
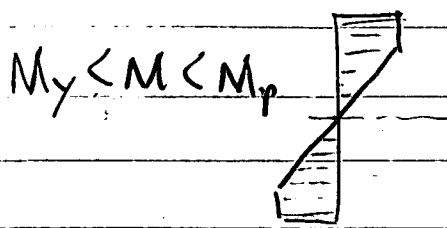




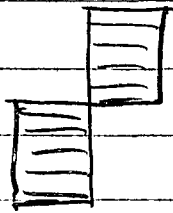
M_y



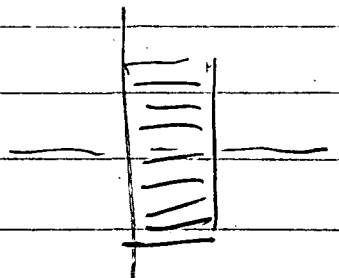
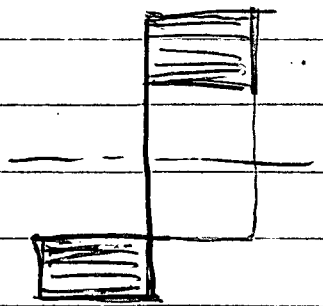
M_{yc}

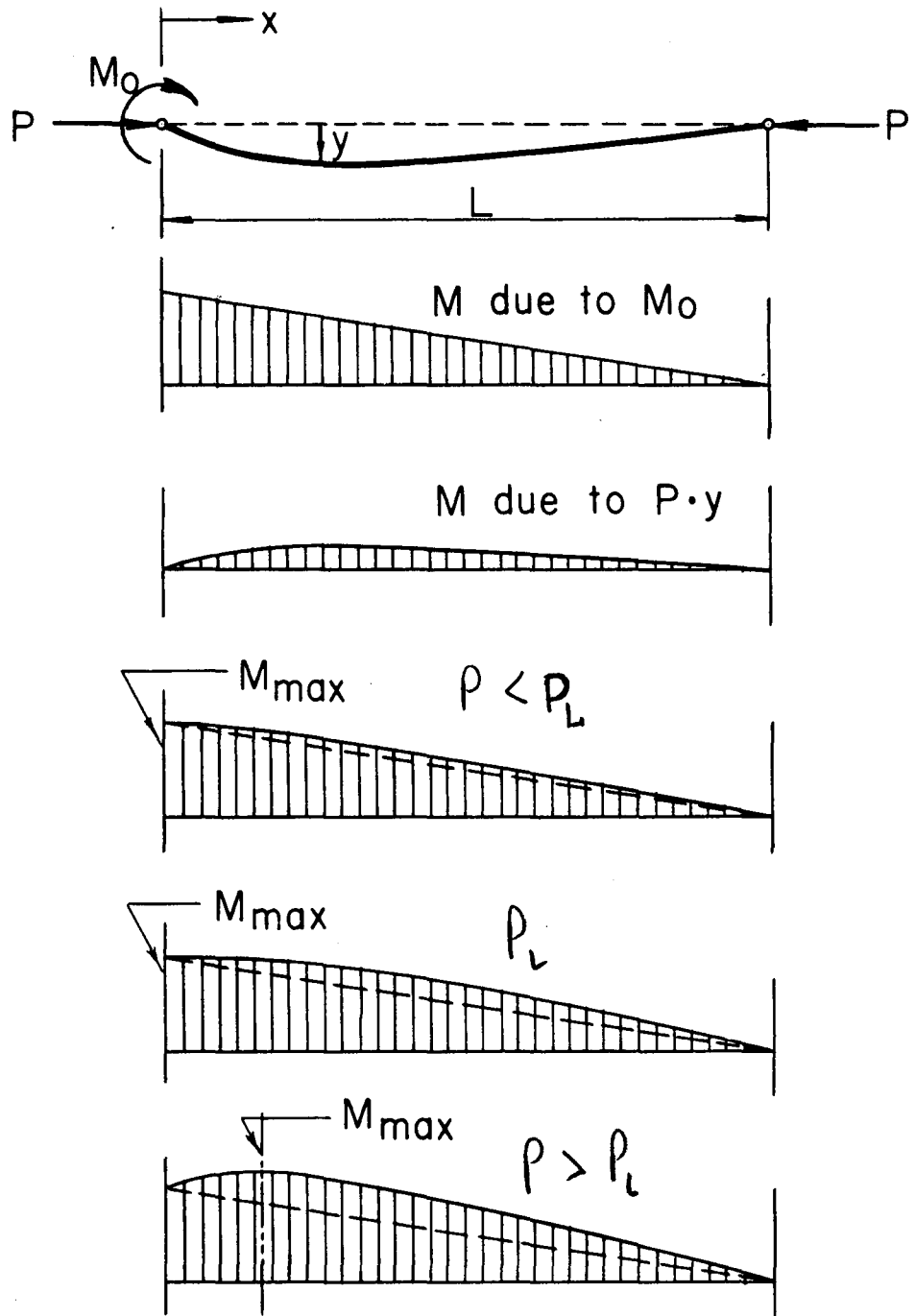


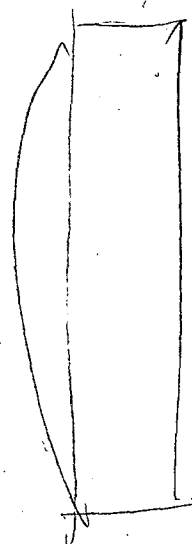
M_p



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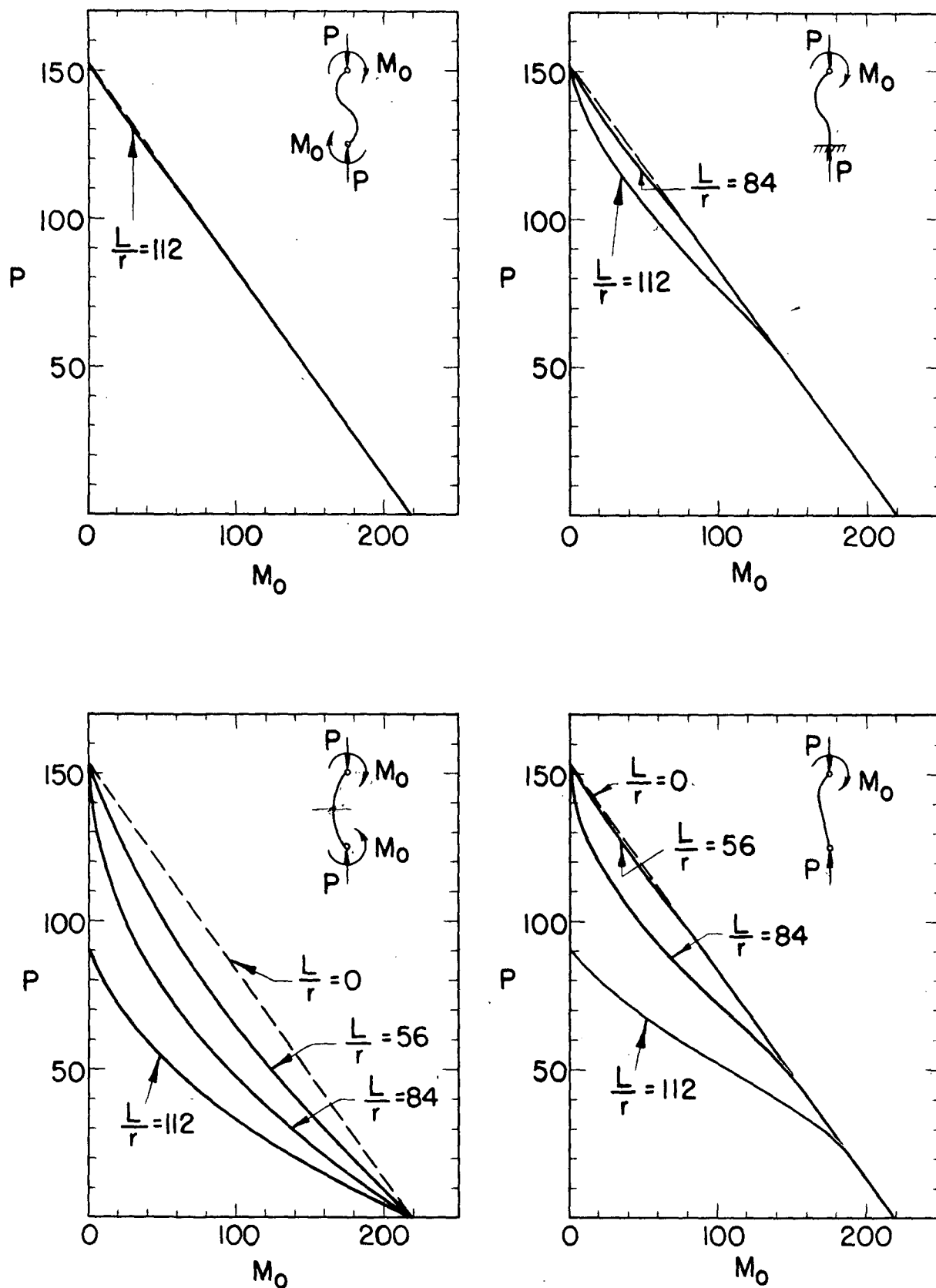


FIG. . INFLUENCE OF SLENDERNESS RATIO ON
INITIAL YIELD INTERACTION CURVE

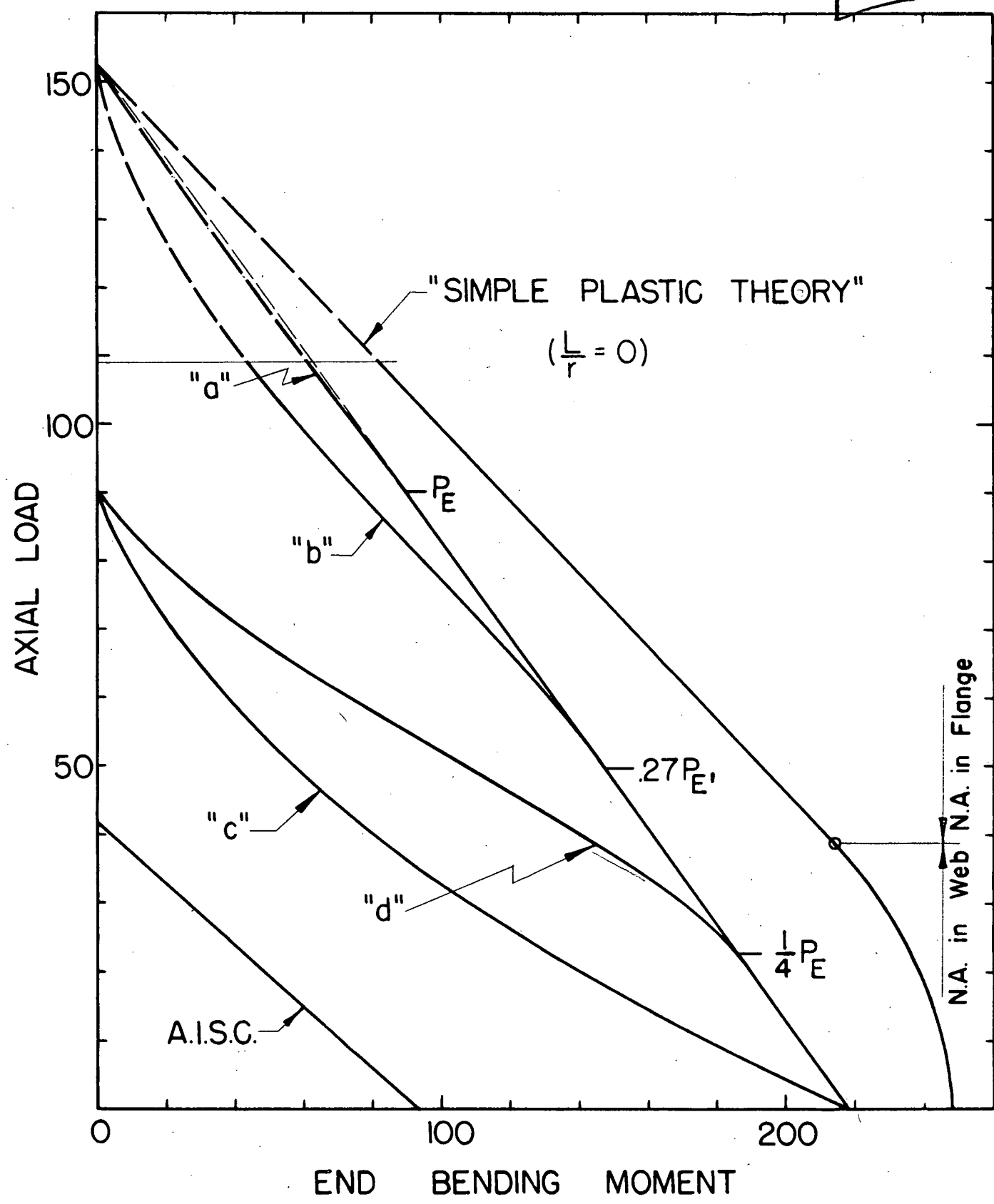
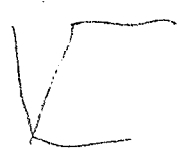


FIG. INFLUENCE OF LOADING CONDITION ON INTERACTION CURVE



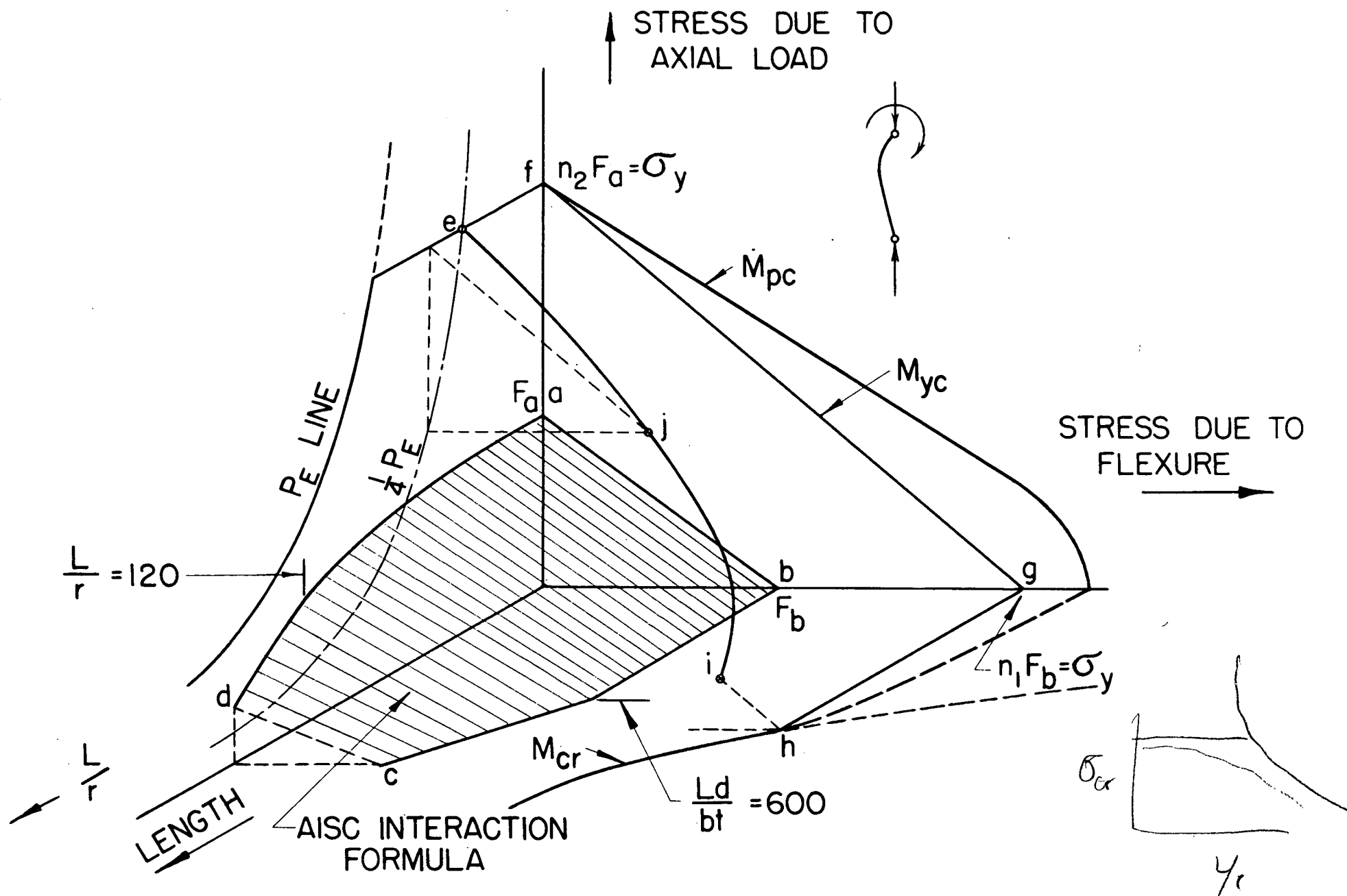
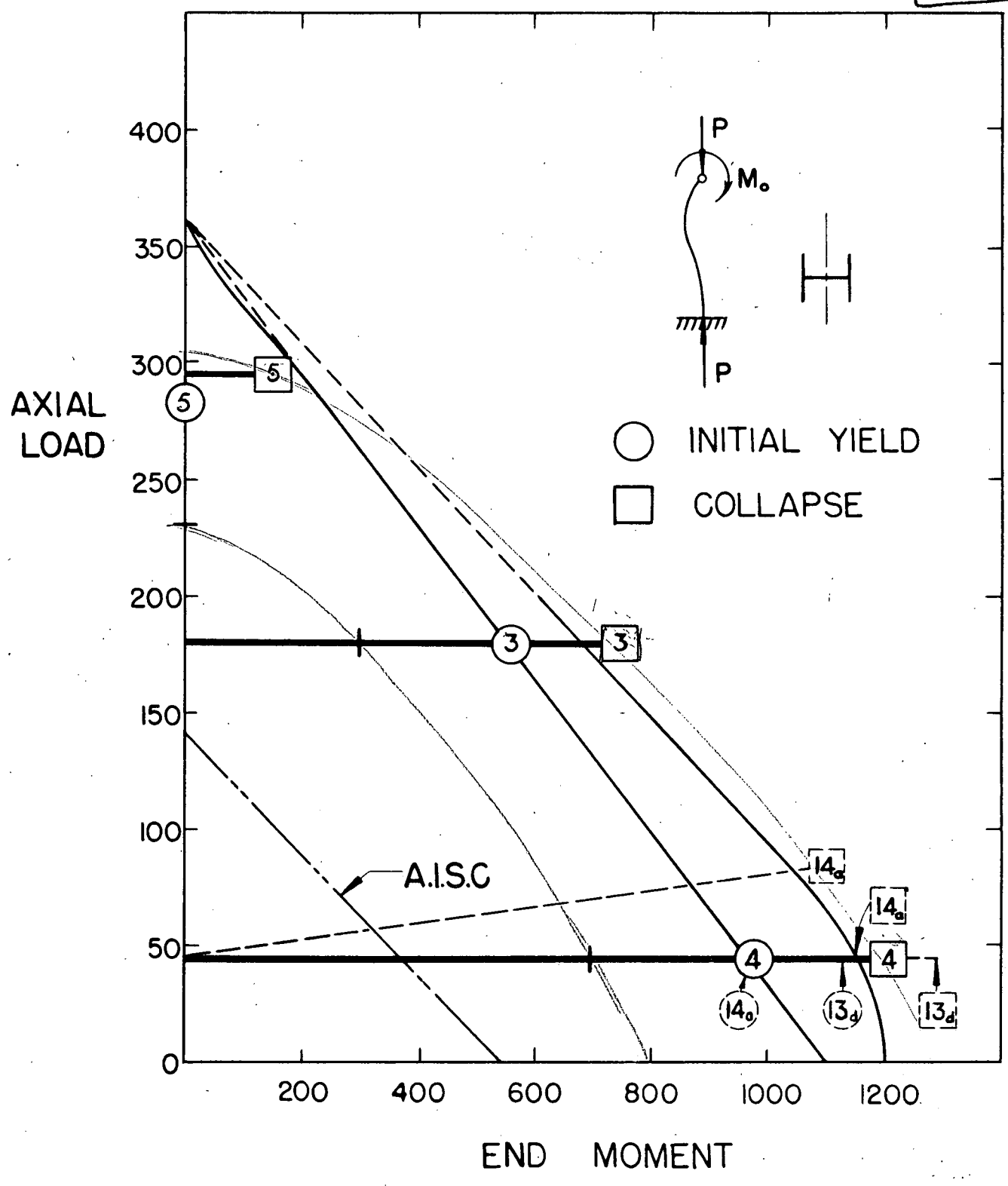
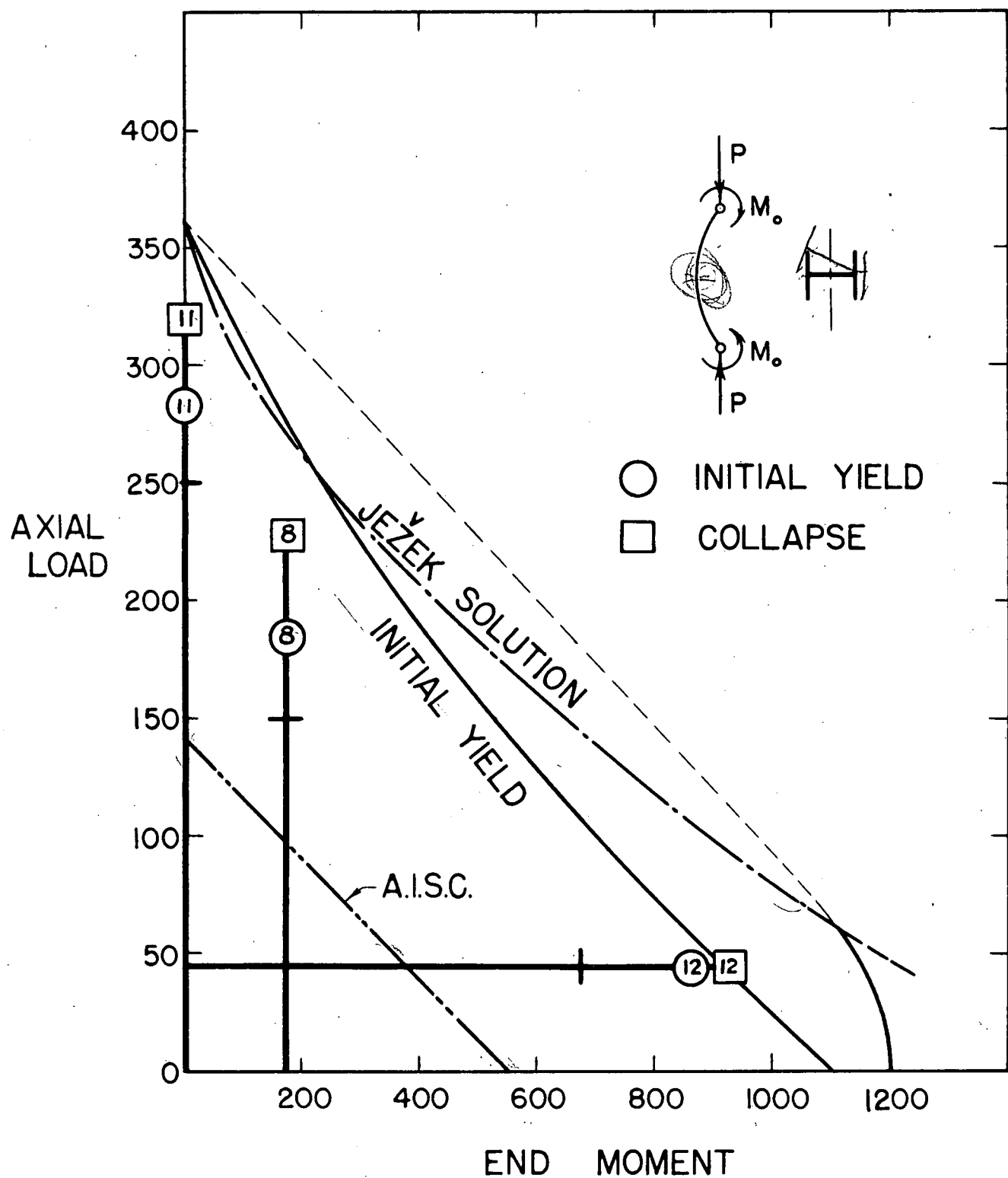


FIG. LOADING CONDITION "d" THREE DIMENSIONAL INTERACTION CURVE

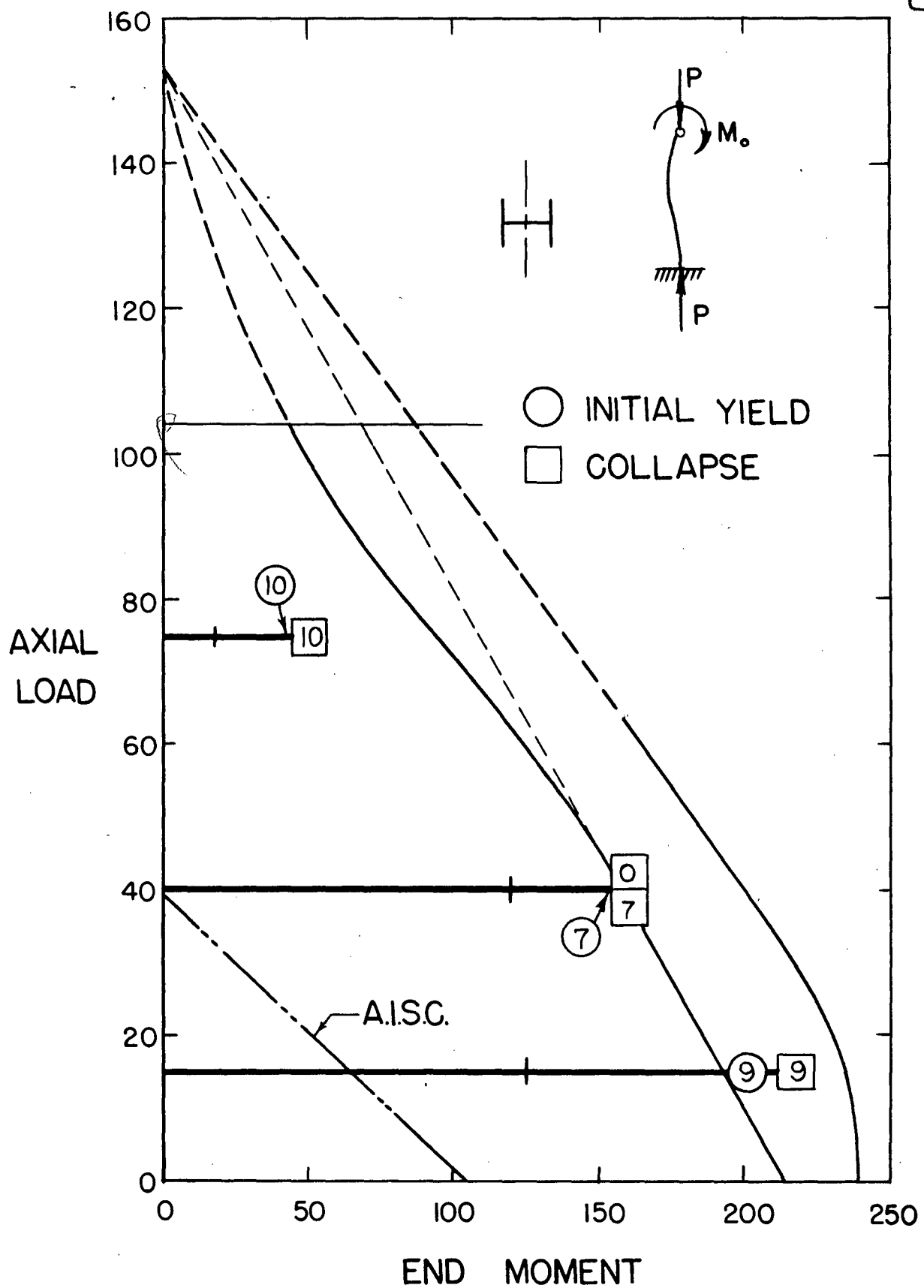


Dr. Johnston





DR. JOHNSTON



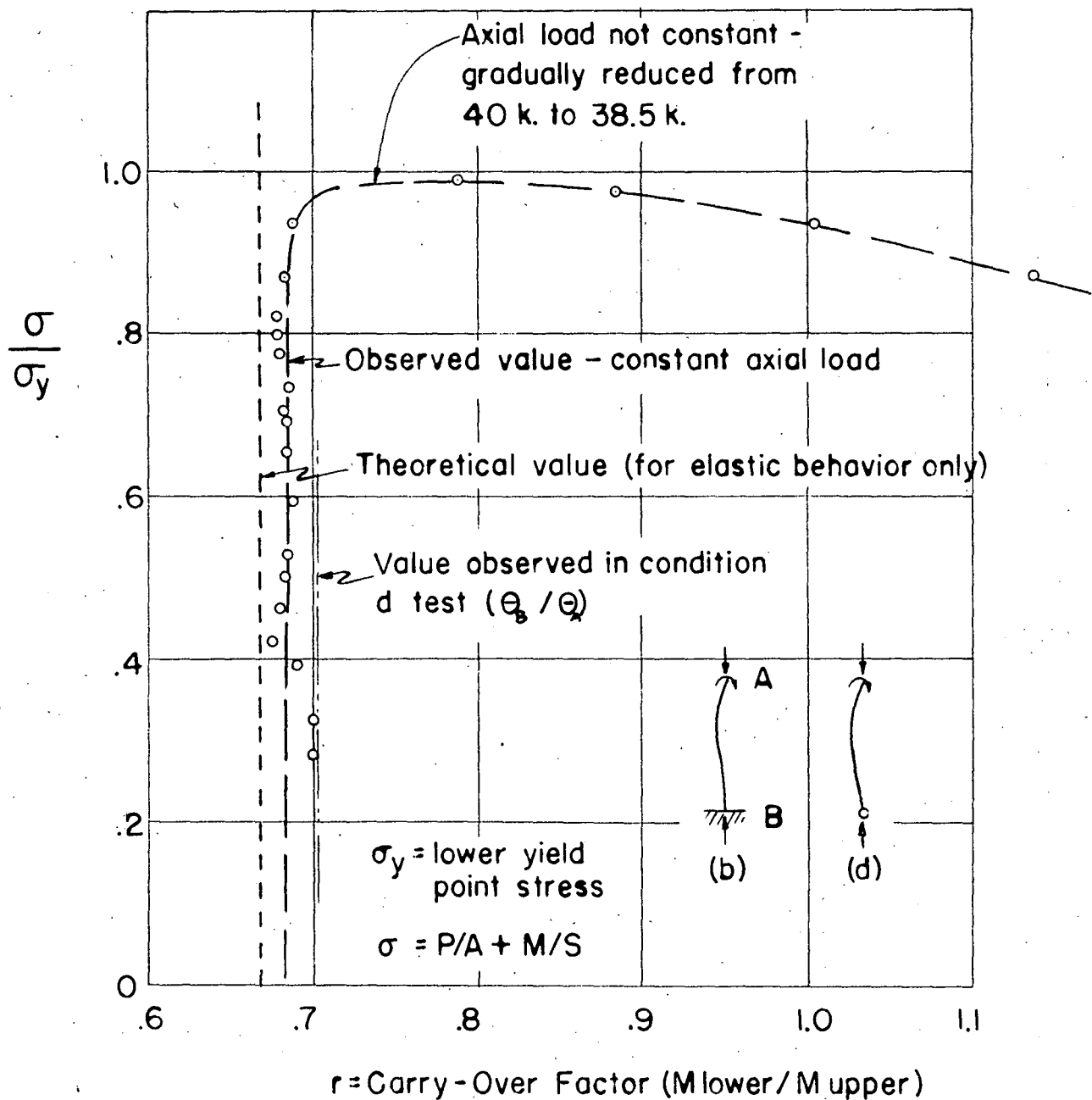
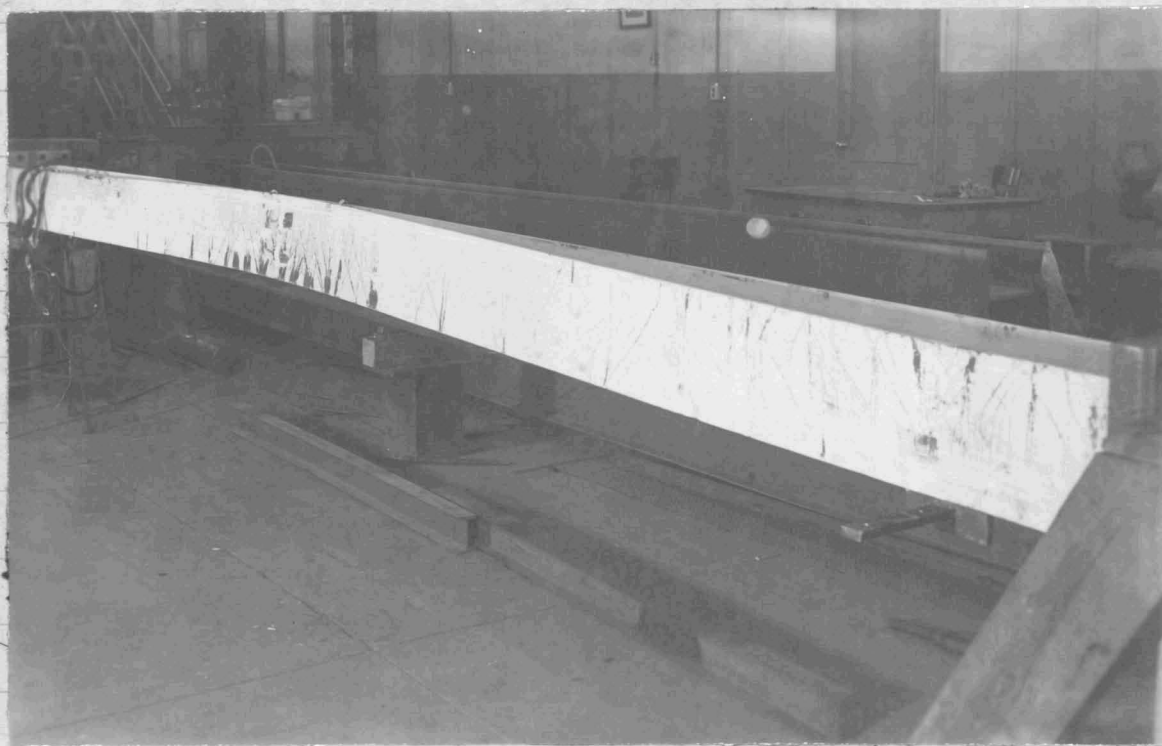


FIG.18.- CARRY-OVER FACTOR; ITS VARIATION WITH STRESS

L7

13



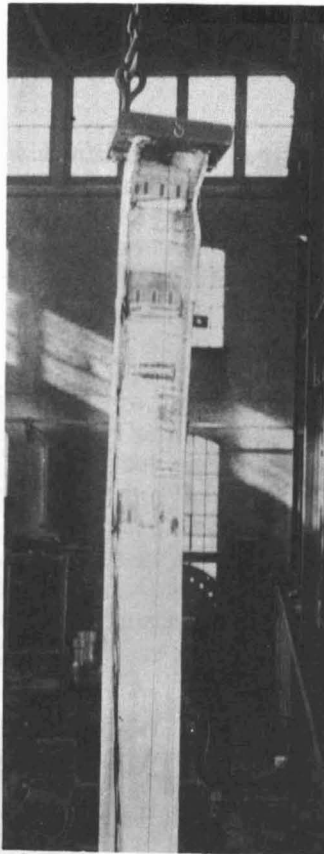


Fig. 10. "Yielding lines" on column after test.

L32
L33
L37

LOCAL BUCKLING OF BEAMS (15)(16)

" " " CONNECTIONS (17)

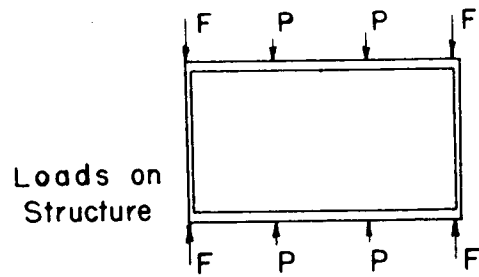
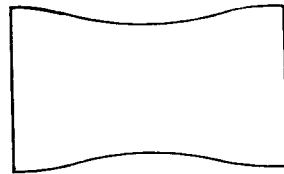
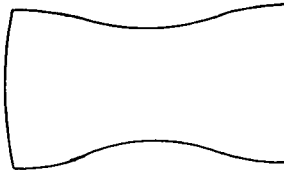


FIG. 16 - BEHAVIOR OF RESTRAINED COLUMNS

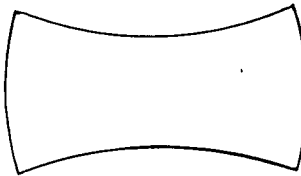
(a)
Fixed End
Beam



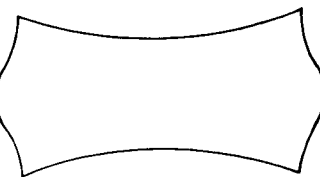
(b)
 $F = 0$
 $P < P_{cr}$



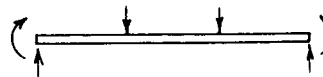
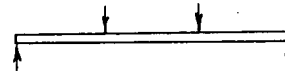
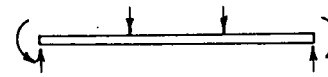
(c)
 $F + P = P_{cr}$



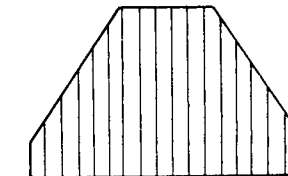
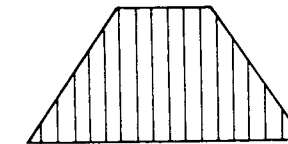
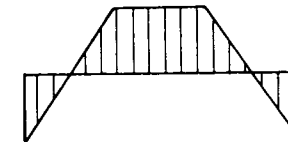
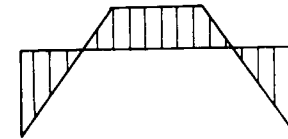
(d)
Near
Collapse



Loading



Moment Diagrams



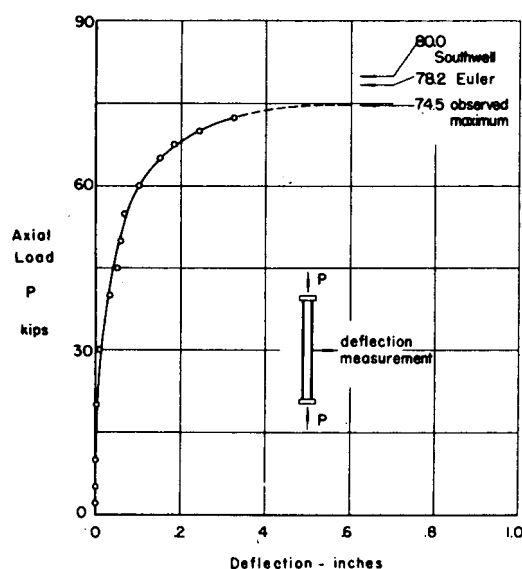


Fig. 15 Column Deflection At Midheight
--pilot test.

ment distribution at any load as well as the buckling load of the entire frame.

Consider the illustrative case shown in Fig. 16*. For an infinitely rigid column, case (a), the beam is "fixed" at the ends. For a more realistic column, case (b), the end moments in the beam are reduced over their "fixed-end" values. As the loads F are applied and increased, case (c), the moment at the ends of the column will decrease and finally become zero at the load which would produce buckling if it were not for the fact that the beams will tend to prevent end rotation. At higher loads, case (d), the column "tries" to buckle but is restrained from doing so by adjoining members. Note that greater than simple beam bending moment is developed at the center of the beam span. Finally, when the combined bending stiffness of the columns and beams framing into a particular joint reaches zero, the entire frame will buckle.

The foregoing procedure of analysis has been restricted principally to the elastic range. One of the purposes of this investigation is to determine to what extent similar procedures are applicable in the inelastic range.

The preceding discussion has been presented as a background for consideration of the experimental methods of obtaining collapse loads and stiffness factors of framed columns.

* The case described is one in which the column is initially deformed in single curvature.

Thus far in the experimental program these loads and factors have been determined for the case where the total axial load is less than P_{cr} (Fig. 16b and 16c). This is also the type of loading acting on single span rigid frames or on the top floors of industrial buildings. In both instances the principal axial force comes from loads in the beams. Consideration of specific frames shows that a condition of negative stiffness would almost never be reached in practice unless additional load is applied directly to the column. Whether or not negative stiffness develops is determined as follows: compute the critical axial load under pin-end conditions for the column; then distribute this load to the beam and determine the maximum moment under simply-supported conditions. The beam will not provide restraint to the column if this moment is greater than the plastic hinge value of the beam.

Although no tests under restrained conditions have been conducted as yet, collapse loads under this type of loading can be determined with the apparatus, simulating different degrees of end restraint (variation in beam stiffness) and different magnitudes of initial load due to bending moment. As was mentioned previously, this is a situation approached in the lower floors of a tier building or in any case where the total load on the compression member exceeds the critical buckling load for a pin ended column.

In England, Professor J. F. Baker and his associates are conducting a program at Cambridge University entitled, "Investigation Into the Behavior of Welded Rigid Frame Structures" under the auspices of the British Welding Research Association. Numerous interim reports have been published by this group, a general report describing the work up to the present time having appeared recently.⁽⁶⁾ The behavior of the restrained column has received particular emphasis in their work and forms a basis for any future studies.

The Interaction Curve

The carrying capacity of the type of member under investigation is a function of applied axial force and end moment. Such an "interaction" curve is shown in Fig. 17. For zero axial load, P , the member is a beam and initial yield and ultimate collapse are determined from elastic and plastic beam theory. For zero moment,